The Life Cycle, Ageing and Longevity

Lecture 1 (of 2): Foundations of Lifecycle Modelling

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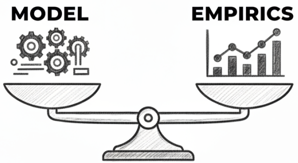
Motivation: why study lifecycle models?

- **Demographic change:** rising longevity, population ageing.
- **Key economic behaviours:** consumption–saving, labour supply, and retirement timing.
- **Policy relevance:** pensions, retirement age, social insurance, taxation, health policy.
- **Today:** stylised facts about lifecycle behaviour; build basic lifecycle model step-by-step
- **Tomorrow:** switch to continuous time, integrate uncertainty (heterogenous agents), analyse “value of prevention”

Taking a step back: why models?

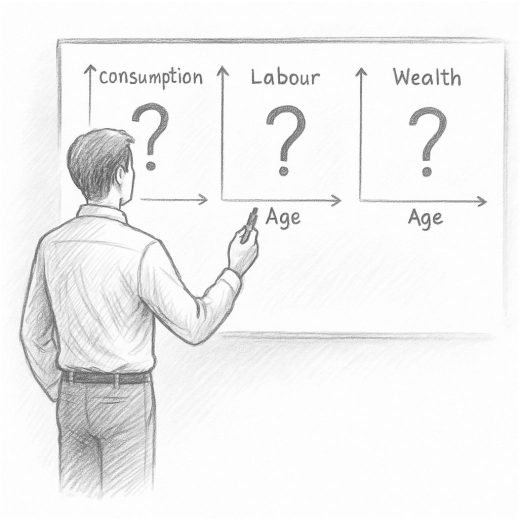
- **Models are tools** used to produce evidence
- **Ultimate goal:** causality - how does y change when x changes holding all else equal
- **Empirical work:** Correlations don't cut it (alternatives: RCTs, natural experiments)
- **Models:** clearly elucidate "mechanisms"
- **Best evidence:** combines model with empirics
  - Empirics: one unit change in $x \rightarrow z$ units change in y
  - Model: *why* changes in x lead to changes in y



"It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts."

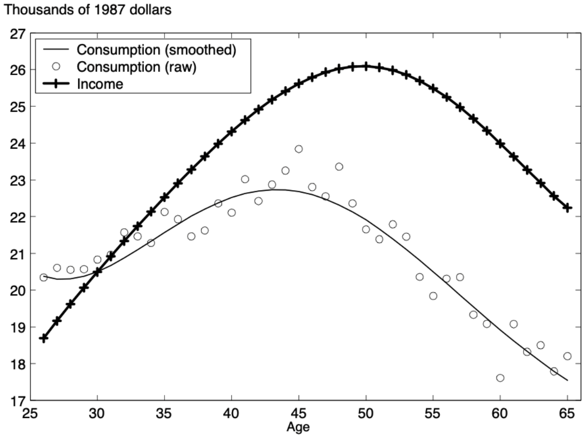
— Sir Arthur Conan Doyle

Empirical patterns over the lifecycle: what do we expect?



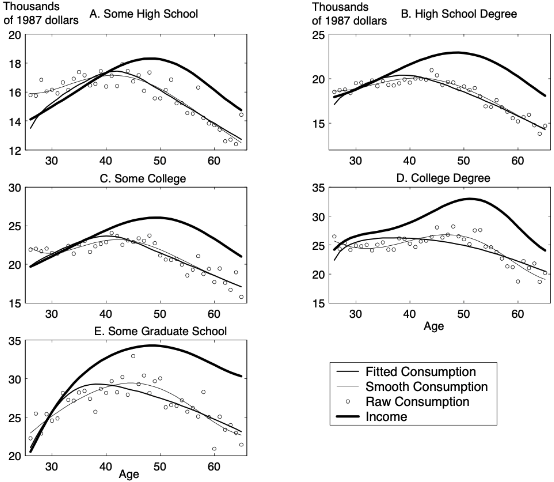
Stylised Fact 1: The consumption-income Hump

- Both income and consumption exhibit a "hump-shape" over the working life.
- Consumption is *smoother* than income, but it is not flat.
- Dissaving early in life (you guys!)
- Followed by period of consumption "tracking" income.



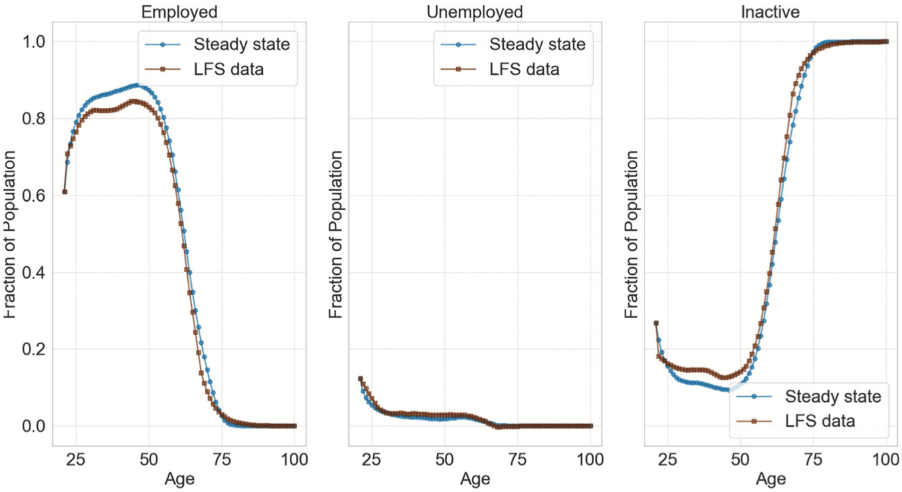
Source: Gourinchas & Parker (2002, Econometrica)

Stylised fact 1: consumption-income hump (by education)



Source: Gourinchas & Parker (2002, Econometrica)

Stylised Fact 2: Labour supply drops of sharply starting at age 60

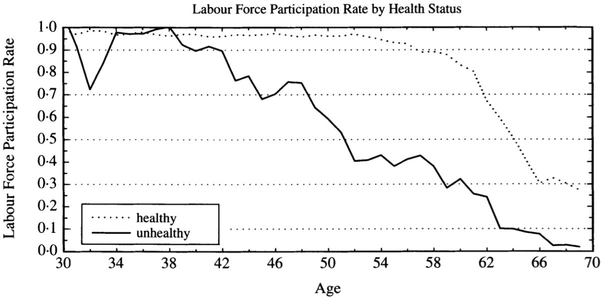


Source: Schindler & Scott (2025, Journal of the Economics of Ageing)

Stylised Fact 3: The role of health

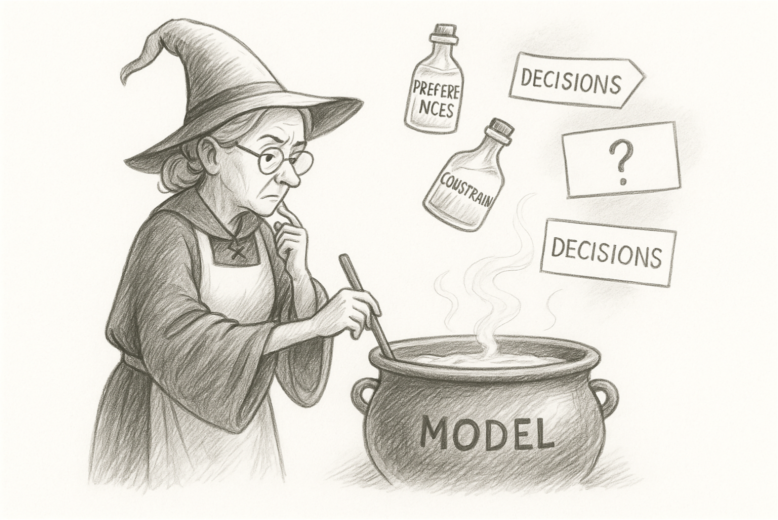
- Health status is a first-order determinant of labour supply

- **Model Link:** Motivates adding *health uncertainty* (shocks) [we do this tomorrow].



Source: French (2005, Review of Economic Studies)

What ingredients should a lifecycle model have?



Apriori key ingredients

- **Preferences:** time-separable utility over consumption and leisure; explicit disutility of work.
- **Constraints:** sequential budget constraints; asset accumulation dynamics.
- **Decisions:** consumption and labour supply each period; *endogenous retirement*
- **Uncertainty:** earnings risk, survival (mortality) risk, health (which may affect utility and/or productivity).
- **Comparative statics & policy:** how longevity, pensions, and taxes shift optimal behaviour and welfare.

Roadmap of rest of lecture

Step 1: Canonical discrete-time consumption–saving model.

Step 2: Add *endogenous labour supply* .

Step 3: Add *absorbing retirement*

Step 4: Add *income, survival, and health shocks* (uncertainty). [tomorrow]

Learning objective

Understand the building blocks and logic needed to analyse ageing and longevity within a lifecycle framework.

Before we start: how to approach this lecture

- **Background knowledge required:**
  - Dynamic programming and Bellman equations
  - Basic understanding of intertemporal optimisation
- **Approach:** somewhat *cookbook-style*
  - I will set up each optimisation problem & briefly discuss how to solve it
  - Then move directly to the key **results and economic insights**
- **Hands-on component:** every model has an accompanying `Jupyter notebook`
  - Solved numerically using simple techniques
  - You can experiment with parameters, explore the code, and see how models like these are actually solved in practice

Household problem:

$$\max_{\{c_t\}_{t=0}^{T}} \quad \sum_{t=0}^{T} \beta^t \, u(c_t)$$

$$\begin{aligned} \text{s.t.} \quad & a_{t+1} = (1+r)a_t + y_t - c_t, \\ & a_{t+1} \geq \underline{a}, \quad a_0 \text{ given}, \\ & a_{T+1} = 0 \quad \text{(terminal condition)}. \end{aligned}$$

- c_t: consumption in period t
- a_t: assets at the beginning of period t
- y_t: income received in period t
- r: constant real interest rate on savings
- β: subjective discount factor $(0 < \beta < 1)$
- $\underline{a}$: borrowing limit (often 0)

Question: what do you think about the terminal condition?

Recursive formulation (finite-horizon problem)

Dynamic programming formulation:

$$V_t(a_t) = \max_{c_t} \left\{ u(c_t) + \beta\, V_{t+1}(a_{t+1}) \right\} \quad \text{s.t.} \quad a_{t+1} = (1+r)a_t + y_t - c_t, \quad a_{t+1} \geq \underline{a}.$$

Boundary condition:

$$V_T(a_T) = \max_{c_T} u(c_T) \quad \text{s.t.} \quad c_T = (1+r)a_T + y_T.$$

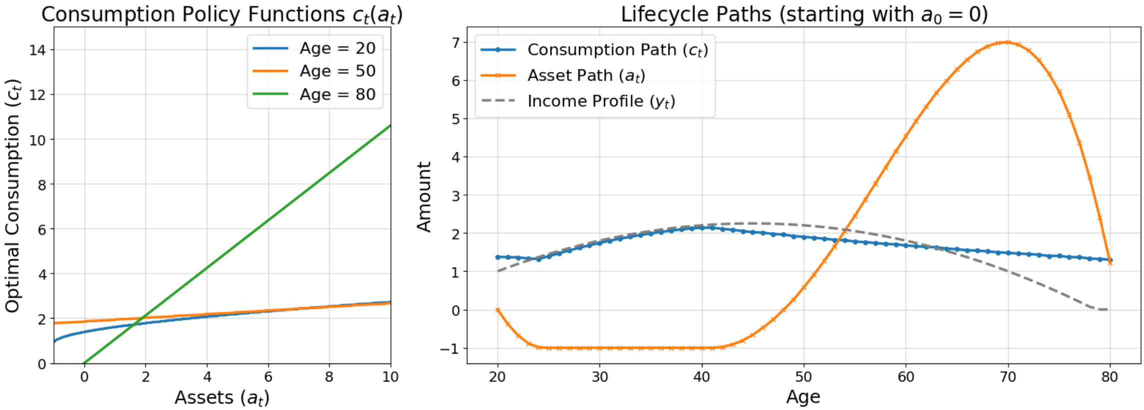
Important:

- The problem can be solved *backwards* from terminal period T.
- V_t depends on t because the horizon shortens as we approach T.

Contrast: infinite horizon

- Stationary environment $\Rightarrow$ drop t index: $V(a)$.
- Policy functions (c_t, a_{t+1}) independent of calendar time.

Canonical consumption-saving model (results)



Adding *endogenous labour supply*

Household problem:

$$\max_{\{c_t,\, h_t\}_{t=0}^{T}} \quad \sum_{t=0}^{T} \beta^t\, u\big(c_t,\ \ell_t\big), \qquad \ell_t \equiv 1 - h_t,$$

$$\begin{aligned} \text{s.t.} \quad & a_{t+1} = (1+r)a_t + w_t z_t h_t - c_t, \\ & a_{t+1} \geq \underline{a}, \quad a_0 \text{ given}, \\ & a_{T+1} = 0 \quad \text{(terminal condition: die with zero assets).} \end{aligned}$$

Interpretation:

- Households now choose both consumption c_t and labour supply h_t each period.
- Utility depends on both consumption and leisure ℓ_t.
- Income is now labour earnings $w_t z_t h_t$ in addition to asset returns.
- Same dynamic structure as before, but with an intratemporal trade-off between leisure and consumption.

Bellman equation & optimality conditions (with labour)

Bellman problem (deterministic):

$$V_t(a_t, z_t) \;=\; \max_{c_t,\, h_t \in [0,1]} \Big\{ u\big(c_t, \ell_t\big) + \beta\, V_{t+1}(a_{t+1}, z_{t+1}) \Big\}$$

$$\text{s.t.} \quad \ell_t = 1 - h_t, \qquad a_{t+1} = (1+r)a_t + w_t z_t h_t - c_t, \qquad a_{t+1} \geq \underline{a}.$$

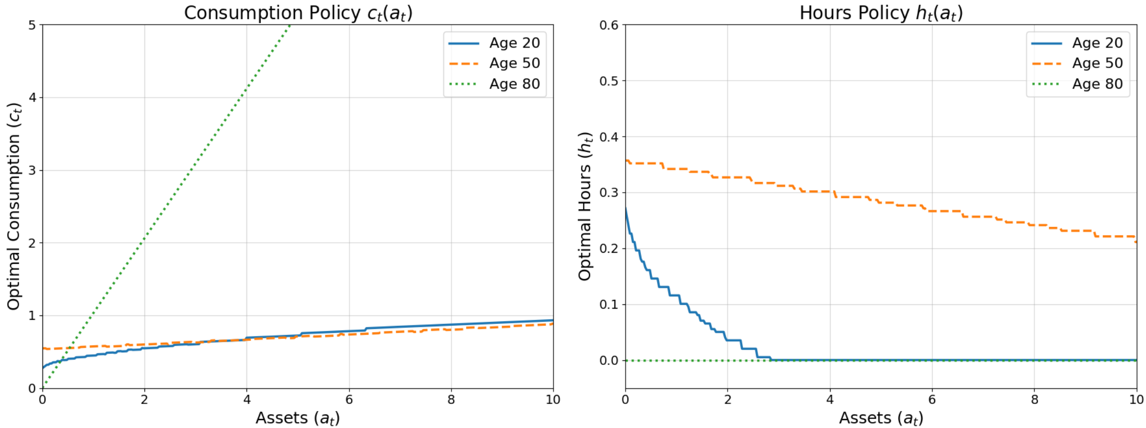
FOCs and envelope (Interior solution):

$$\begin{aligned}
\text{(Euler / intertemporal)} \quad u_c(c_t, \ell_t) \;&=\; \beta(1+r)\, u_c(c_{t+1}, \ell_{t+1}) \\
\text{(Intratemporal)} \quad u_\ell(c_t, \ell_t) \;&=\; u_c(c_t, \ell_t)\, w_t z_t
\end{aligned}$$

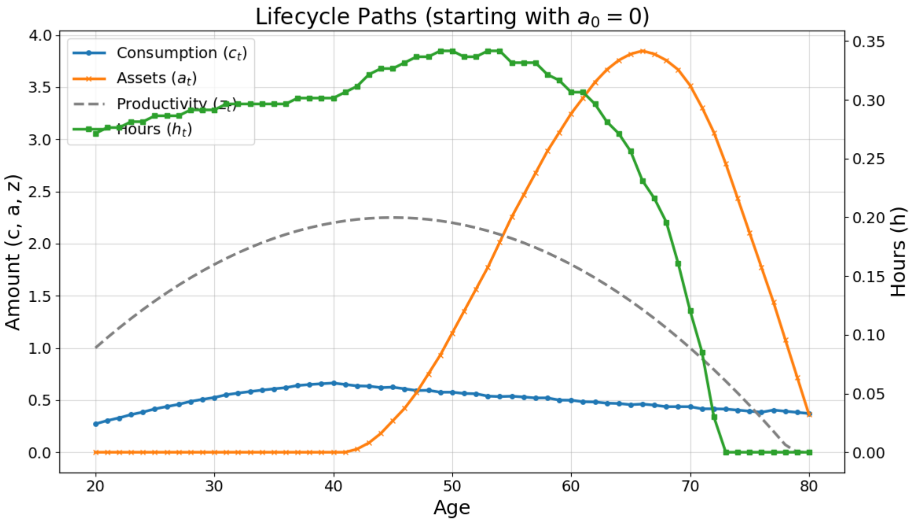
Handling constraints (KKT conditions):

$$\begin{aligned}
\text{Saving } (a_{t+1} = \underline{a})\text{:} \quad u_c(c_t, \ell_t) \;&>\; \beta(1+r)\, u_c(c_{t+1}, \ell_{t+1}), \\
\text{Labour } (h_t = 0)\text{:} \quad u_\ell(c_t, \ell_t) \;&>\; u_c(c_t, \ell_t)\, w_t z_t \\
\text{Labour } (h_t = 1)\text{:} \quad u_\ell(c_t, \ell_t) \;&<\; u_c(c_t, \ell_t)\, w_t z_t
\end{aligned}$$

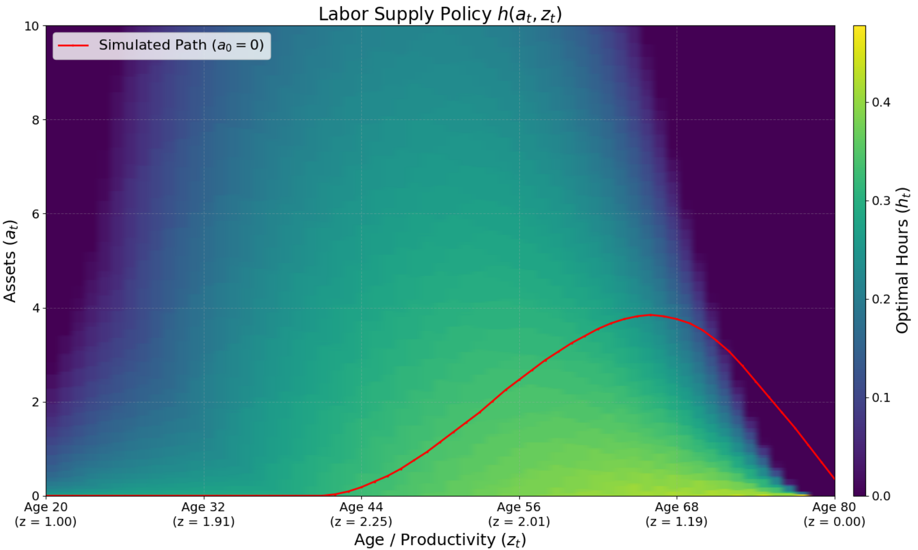
Lifecycle model with endogenous labour supply (policy functions)



Lifecycle model with endogenous labour supply (results)



Policy function heatmap



Adding *absorbing retirement*: household problem

Bellman problem:

$$V_t(a_t, z_t, R_t) = \max_{\substack{c_t \geq 0,\ h_t \in [0,1], \\ d_t \in \{0,1\}}} \left\{ u\big(c_t,\ \ell_t\big) + \beta\, V_{t+1}\big(a_{t+1}, z_{t+1}, R_{t+1}\big) \right\}$$

Constraints:

$$\ell_t = 1 - (1 - R_t)\, h_t \qquad \text{(full leisure if retired)}$$

$$a_{t+1} = (1 + r)a_t + (1 - R_t)\, w_t z_t h_t + R_t\, b - c_t, \qquad a_{t+1} \geq \underline{a}$$

Interpretation

- Retirement choice d_t is only relevant while working ($R_t = 0$); once retired, $R_{t+s} = 1$ forever.
- Income while working: $w_t z_t h_t$; while retired: b and $h_t = 0$.
- Leisure is full in retirement; during work, it trades off with earnings via h_t.

Stopping time problem: coupled Bellman equation system

$$\boxed{V_t(a_t, z_t, 0) \;=\; \max\Big\{ \underbrace{V_t^W(a_t, z_t)}_{\text{keep working}},\; \underbrace{V_t^R(a_t, z_t)}_{\text{retire now}} \Big\}.}$$

$$V_t^W(a_t, z_t) = \max_{c_t \ge 0,\, h_t \in [0,1]} \Big\{ u(c_t,\, 1 - h_t) + \beta\, V_{t+1}(a_{t+1}, z_{t+1}, 0) \Big\}$$

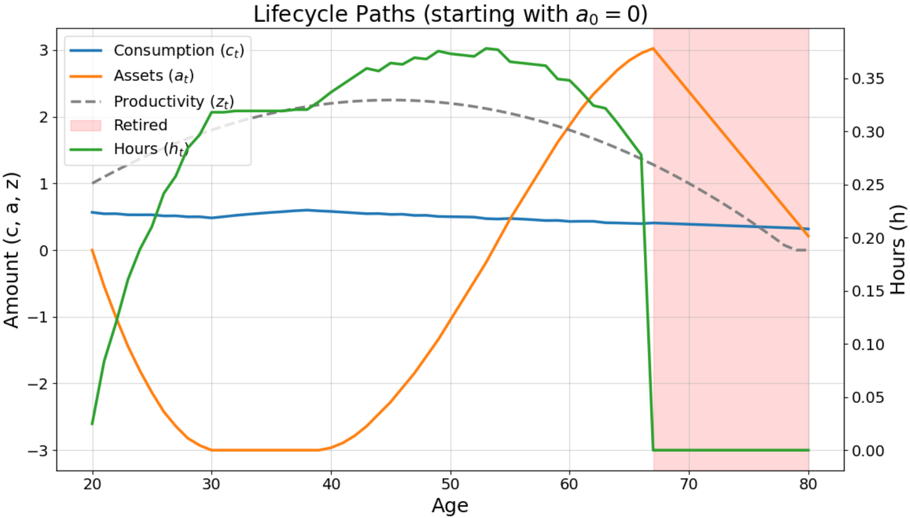
$$\text{s.t.} \quad a_{t+1} = (1 + r)a_t + w_t z_t h_t - c_t, \qquad a_{t+1} \ge \underline{a}.$$

$$V_t^R(a_t, z_t) = \max_{c_t \ge 0} \Big\{ u(c_t,\, 1) + \beta\, V_{t+1}(a_{t+1}, z_{t+1}, 1) \Big\}$$

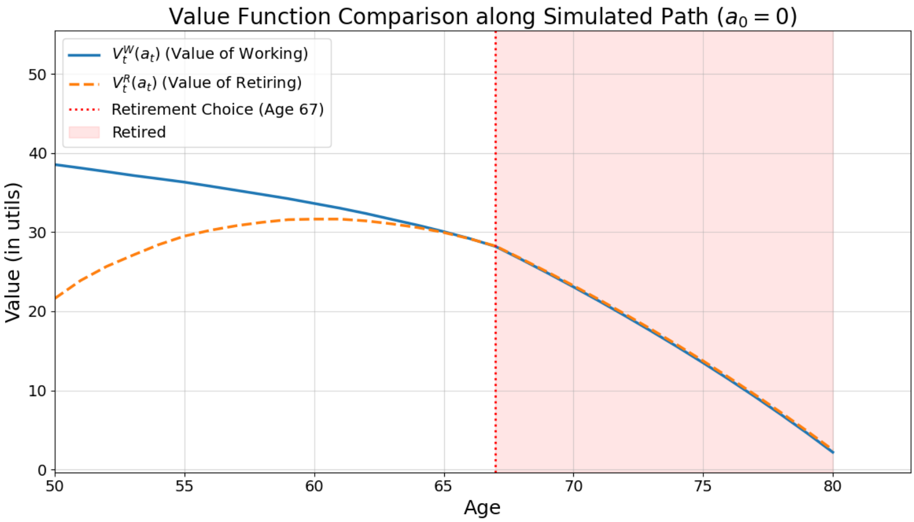
$$\text{s.t.} \quad a_{t+1} = (1 + r)a_t + b - c_t, \qquad a_{t+1} \ge \underline{a}.$$

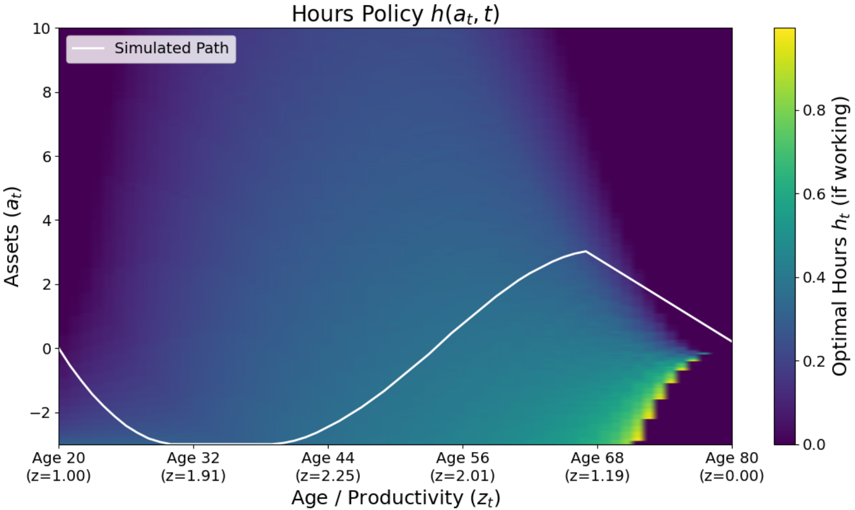
Notes

- When $R_t = 1$, retirement is *absorbing*: $h_t = 0$ and income is the fixed pension b.
- The **retirement region** is the set of states where $V_t^R(a_t, z_t) \ge V_t^W(a_t, z_t)$.

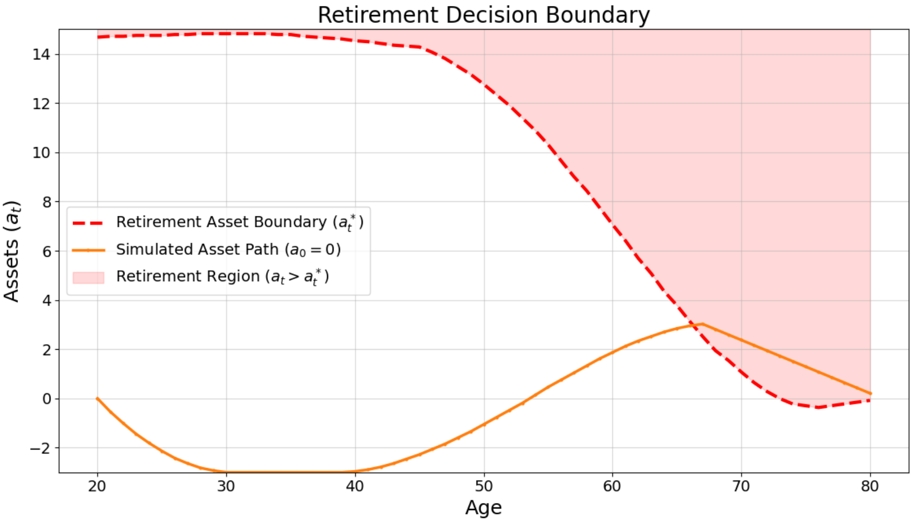

Lifecycle Paths (starting with $a_0 = 0$)
Consumption (c_t)
Assets (a_t)
Productivity (z_t)
Retired
Hours (h_t)
Amount (c, a, z)
Hours (h)
Age
3
2
1
0
−1
−2
−3
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
20
30
40
50
60
70
80

Optimal stopping time: relative value of continuing to work and retiring




Hours Policy $h(a_t, t)$
Simulated Path
Assets (a_t)
10
8
6
4
2
0
−2
Age 20 (z=1.00)
Age 32 (z=1.91)
Age 44 (z=2.25)
Age 56 (z=2.01)
Age 68 (z=1.19)
Age 80 (z=0.00)
Age / Productivity (z_t)
Optimal Hours h_t (if working)
0.8
0.6
0.4
0.2
0.0

The retirement boundary by age and assets



Summary & preview: lifecycle models

Key takeaways from today

- **Motivation:** lifecycle models are essential for analysing consumption, saving, labour supply, and retirement decisions in ageing societies.
- **Building blocks:** preferences, constraints, uncertainty, and policy parameters jointly determine optimal behaviour.
- **Model structure:** we moved from the canonical consumption–saving model $\rightarrow$ endogenous labour $\rightarrow$ optimal stopping time / absorbing retirement.
- **Core insight:** behaviour over the life course reflects intertemporal trade-offs between consumption, leisure.

Tomorrow: Lecture 2 — uncertainty & continuous Time

- Introduce **earnings, survival, and health shocks.**
- Model can be used to study effects of **longevity and health** on lifecourse behaviour.
- Analyse the **"value of prevention"** and implications of health shocks.